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ORIGINAL ARTICLE

An investigation of vendor-managed inventory application in supply chain: the EOQ model with shortage

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Abstract Recent researches have shown the importance of improving the supply chain competitiveness by means of strategic alliances. This study considers the retailer–supplier partnership through a vendor-managed inventory (VMI) system and develops an analytical model to explore the effect of important supply chain parameters on the cost savings realized from collaborative initiatives. This model is developed for a two-level supply chain consisting of a single supplier and a single retailer and examines the inventory management practices before and after implementation of VMI. The results of analytical examination show that VMI implementation in economic order quantity model when shortage is backlogged sometimes has the ability to reduce total costs of supply chains. Three numerical examples are also given to support this claim.

Keywords Vendor-managed inventory · Supply chain management · Economic order quantity · Shortage · Backlog

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1 Introduction

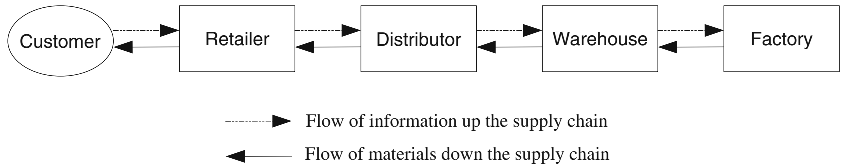
A supply chain is a dynamic system that includes all activities involved in delivering a product from the stage of raw material to the customer. These activities include manufacturing, inventory control, distribution, warehousing, and customer service. Supply chain management coordinates and integrates all of these activities into a smooth process.

The main objective of the supply chain management is to minimize system-wide costs while satisfying service-level requirements. With increasing global competition and emergence of e-business, supply chain management is viewed as a major solution to cost reduction and profitability [21].

Inventory has been considered one of the major drivers of the supply chain and its management [7]. "There is nothing more important within the realm of supply chain management than the management of inventory…" [18]. As such, decisions regarding inventory replenishment have a direct effect on supply chain performance.

In a traditional supply chain, each partner (company) is responsible for its own inventory control and production or distribution ordering activities, and the interactions between partners are limited to the feed-forward physical flow of products and the feedback flow of information. One fundamental characteristic and problem that all partners of a traditional supply chain (such as retailers, distributors, manufacturers, raw material suppliers) must solve is "just how much to order the production system to make (or the suppliers to supply) to enable a supply chain echelon to satisfy its customers' demands". This is the classic production/inventory control problem [9]. A simple schematic of a four-echelon supply chain, comprising retailer, distributor, warehouse, and factory, is illustrated in Fig. 1 [10].

Fig. 1 A traditional supply chain



In a vendor-managed inventory (VMI), besides the traditional supply chain information and material flows, the supplier controls the retailer's inventory level so as to ensure that desirable customer service levels are maintained. In other words, VMI has been described as an inventory and supply chain management tool in which the supplier has taken the responsibility for making decisions as to the timing and amounts of inventory replenishment [10].

Evidence has shown that VMI can improve supply chain performance by decreasing inventory levels and increasing fill rates; as a result, industry use of VMI has grown over time [12]. VMI is a collaborative commerce initiative that integrates operations between suppliers and retailers through information sharing and business process reengineering. By the aid of information technologies, such as electronic data interchange (EDI) or internet-based XML protocols, retailers can share sales and inventory information with suppliers on a real-time basis. A simple diagram of a VMI supply chain is found in Fig. 2 [9].

Suppliers can then use this information to plan production runs, schedule deliveries, and manage order volumes and inventory levels at the retailer's stock-keeping facilities [26].

The potential benefits from VMI are very compelling and can be summarized as reduced inventory costs for the supplier and retailer and improved customer service levels, such as reduced order cycle times and higher fill rates [1, 24]. These benefits have been realized by successful retailers and suppliers, most notably Wal-Mart and key suppliers like Proctor & Gamble [4].

In order to determine the impact of important supply chain parameters on the cost savings to be realized from collaborative initiatives such as VMI, in this paper, the previous research is extended and an analytical model is developed for an economic order quantity (EOQ) problem where shortage is backlogged. The model is considered for a two-level supply chain consisting of a single supplier (vendor or distributer) and a single buyer (retailer) to examine the inventory management practices before and after the VMI implementation. These results have managerial ramifications in that they can help demonstrate when and by how much VMI is likely to produce benefits for supplier–buyer dyads in practice.

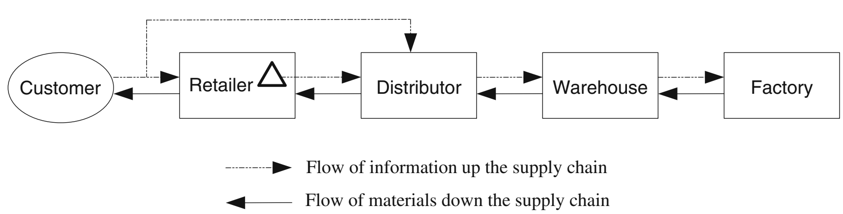
The rest of the paper is structured as follows: in the next section, a review of the literature on VMI is presented. While section 3 presents the framework of the proposed model, section 4 contains the model along with its assumptions. Section 5 uses the framework to analyze cost savings due to VMI. Three numerical examples are presented in section 6 to justify and demonstrate the application of the proposed method. Finally, conclusions and recommendations for future research are presented in section 7.

2 Literature review

An early conceptual framework of VMI was described by Magee [19] when discussing who should have authority over the control of inventories. However, interest in the concept has only really developed during the 1990s. Companies have looked to improve their supply chains as a way of generating a competitive advantage, with VMI often advocated. This strategy has been particularly popular in the grocery sector but has also been implemented in sectors as diverse as steel, books, and petrochemicals [10]. Examples of the companies that popularized VMI in 1980s are Wal-Mart, K-Mart, and Proctor & Gamble [2, 24].

The use and the benefits of VMI were originally documented by a few authors with the primary research being conducted by Blatherwick [2], Cachon and Fisher [3], Clark and Hammond [8], Fraza [13], and Waller et al. [24]. In all documented cases of organizations that use VMI, there was some connection between the members in order to facilitate the exchange of information on inventory levels, product usage, and re-supply issues. Generally, this connection was provided with EDI [12]. Haavik [14] stated

Fig. 2 A VMI supply chain



that EDI tools were needed to realize the full benefits of VMI. Lawrence and Vokurka [17] and Challener [5], however, described situations where the exchange was a manual process. In these cases, a representative from the upstream member physically monitored the inventory level. In order to illustrate the potential inventory cost savings from integrated inventory management, early researchers studied simple supply chains with a single supplier and a single retailer.

Woo et al. [25] and Yu and Liang [27] extended the two-echelon inventory supply chains to three echelon ones where the supplier was a manufacturer and his raw materials' inventory was involved. In order to streamline the supply chain, the supplier expected to synchronize his production cycles with his retailers' orderings. The supplier and his retailers employed a common replenishment cycle policy to reduce supply chain inventory cost. Vigtil [23] described a set of five case studies that indicated sales forecasts and inventory positions were the most valuable information provided to suppliers by the retailers in a VMI relationship.

Generally speaking, the VMI research can be classified into two groups:

(a) Strict managerial and functional subjects along with simulation procedure (see, for example, [9, 15, 21])
(b) Analytic and mathematical terms and presentations of the economic analysis (see, for example, [6, 11, 16, 20, 22, 26])

Within the second group, Dong and Xu [11] presented an analytical model to evaluate the short-term and long-term impact of VMI on supply chain profitability by analyzing the inventory systems of the parties involved. The impact of VMI was also compared with that of full channel coordination, and it was found that, in the short term, VMI can accomplish what full channel coordination is set to accomplish. They formulated appropriate mathematical models for a buyer–supplier channel structure, examined the effects of a VMI strategy on the various cost components of both parties, and then analyzed the role of VMI in a supply chain initiative. In particular, the effects of an integrative VMI program on total relevant costs and profits were investigated. Several common assumptions that also were used in their inventory-channel coordination research were: the inventory system of the buyer can be described by an EOQ policy, the demands are deterministic, there are no stock-outs, and the lead times are also deterministic.

Yao et al. [26], using the same assumptions as Dong and Xu's [11], along with an additional assumption (the order quantity for the supplier is likely to be an integer multiple of the buyer's replenishment quantity), presented an analytical model to determine how key logistics parameters, most notably ordering costs and inventory carrying charges, can affect the benefits to be derived from VMI. They then determined how the benefits were likely to be distributed between a buyer and a supplier in a supply chain, given these logistics parameters. Finally, they presented a numerical example to illustrate their results.

Van Der Vlist et al. [22] extended the Yao et al. [26] model along with the costs of shipments from the supplier to the buyer. Sofifard et al. [20] presented an analytical model for a single-buyer–single-supplier model to explore the effects of collaborative supply-chain initiatives such as VMI with the economic production quantity (EPQ) manner. Furthermore, Jasemi [16] developed a supply chain model with single-supplier n-buyers and compared the VMI system with traditional types. He also made a pricing system for profit-sharing between parties.

It should be noted that, in all of the previous researches within the second group, the EOQ model with infinite production rate was considered.

3 Modeling framework

In order to determine the impact of important supply chain parameters on the cost savings to be realized from collaborative initiatives such as VMI, consider a two-level supply chain consisting of a single supplier and a single retailer for which we examine the inventory management practices before and after the implementation of VMI. Although many of the results can be generalized to more complex supply chains, a simple supply chain is used for computational ease. We assume that the retailer faces external demand from consumers.

In a supply chain without VMI on the one hand, the supplier observes consumer demand only indirectly through the retailer's ordering policy. In fact, the retailer company appears to be the “leader” in this relationship and the supplier just takes the order quantity from the retailer and makes the necessary delivery. In other words, the supplier does not have any responsibility for the production holding. On the other hand, in a supply chain with VMI, the retailer no longer manages its inventory system and leaves it to the supplier to determine inventory levels, order quantities, lead times, etc. [11].

In a supply chain with VMI, the supplier's information system directly receives consumer demand data. As a result, the supplier now has the combined inventory with order setup and holding cost [11]. The supplier with regard to its own inventory cost, which equals the total cost of the supply chain, determines the timing and the quantity of production in every cycle. The major difference between not using and using VMI is that the retailer's order quantity is determined by the supplier in a VMI system [26].

Figure 3 presents the modeling framework of VMI and non-VMI supply chains, where the triangles show the stores.

4 The EOQ policy when shortage is backlogged

In this section, the assumptions and the notations are first defined. Then, the total cost of both the traditional (non-VMI) and the integrated (VMI) supply chains are modeled.

4.1 Assumptions

The mathematical model of this research will be developed on the basis of the following assumptions:

(a) A single-supplier–single-buyer supply chain with one item is considered.
(b) Shortage is allowed and is completely backlogged ($\hat{\pi} \neq 0$ and $\pi = 0$).
(c) Deliveries of orders are assumed to be instantaneous, that is, the lead time is zero.
(d) Costumer's demand is deterministic.
(e) The production rate is infinite.
(f) The product price is fixed in the planning period.

4.2 Notations

The following notations will be used to develop the proposed model:

TC_{noVMI}	Total costs of non-VMI supply chain
TC_{VMI}	Total costs of VMI supply chain
A_S	Supplier's ordering cost per unit
A_B	Buyer's ordering cost per unit
Q_{noVMI}	Order quantity of non-VMI chain
Q_{VMI}	Order quantity of VMI chain
D	Buyer's demand rate
h_B	Product's carrying cost per unit held in buyer's store in a period
KB_0	Buyer's inventory cost before VMI
KB_1	Buyer's inventory cost after VMI
KS_0	Supplier's inventory cost before VMI
KS_1	Supplier's inventory cost after VMI
b	Fixed amount of shortage in a cycle
π	Fixed cost of shortage per unit
$\hat{\pi}$	Cost of shortage per unit per time

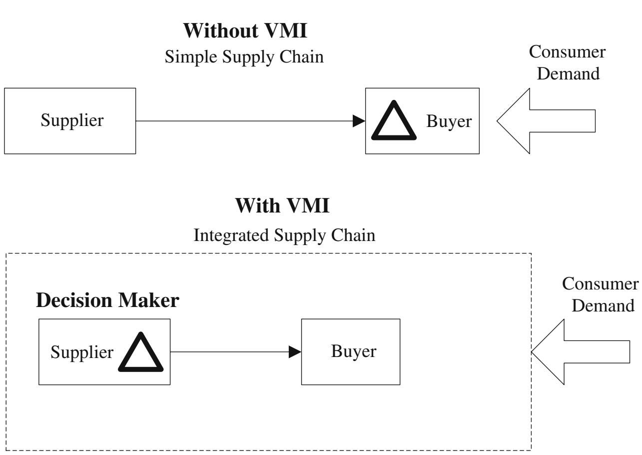


Fig. 3 The modeling frameworks

4.3 The case of non-VMI (traditional) supply chain

Prior to implementing VMI, the inventory cost of the buyer and the supplier and, hence, the total inventory costs of the supply chain are calculated as follows:

$$KB_0 = \frac{A_B D}{Q_{noVMI}} + \frac{h_B}{2Q_{noVMI}}(Q_{noVMI} - b)^2 + \frac{\hat{\pi} b^2}{2Q_{noVMI}} + \frac{\pi b D}{Q_{noVMI}} \quad (1)$$

$$KS_0 = \frac{A_S D}{Q_{noVMI}} \quad (2)$$

$$TC_{noVMI} = KB_0 + KS_0 = \frac{A_B D}{Q_{noVMI}} + \frac{h_B}{2Q_{noVMI}}(Q_{noVMI} - b)^2 + \frac{\hat{\pi} b^2}{2Q_{noVMI}} + \frac{\pi b D}{Q_{noVMI}} + \frac{A_S D}{Q_{noVMI}} \quad (3)$$

The buyer's inventory cost in Eq. 1 is a function of both Q_{noVMI} and b for which the optimal values can be obtained by partial derivatives. Differentiating Eq. 1 with respect to Q_{noVMI} and setting it zero results in:

$$\frac{\partial KB_0}{\partial Q_{noVMI}} = 0 \quad (4)$$

$$\Rightarrow -\frac{1}{Q_{noVMI}^2}\left(DA_B + \frac{1}{2}h_B(Q_{noVMI} - b)^2 + \pi D b + \frac{1}{2}\hat{\pi} b^2\right) + \frac{h_B}{Q_{noVMI}}(Q_{noVMI} - b) = 0 \quad (5)$$

$$\Rightarrow -\frac{1}{2}(Q_{noVMI} - b)^2 + Q_{noVMI}(Q_{noVMI} - b) = \frac{1}{h_B}\left(DA_B + \pi D b + \frac{1}{2}\hat{\pi} b^2\right) \quad (6)$$

$$\Rightarrow \frac{1}{2}Q_{noVMI}^2 = \frac{1}{h_B}\left(DA_B + \pi D b + \frac{1}{2}\hat{\pi} b^2\right) + \frac{1}{2}b^2 \quad (7)$$

At the same time, differentiating Eq. 1 with respect to b results in:

$$\frac{\partial \mathrm{KB}_0}{\partial b}=0 \tag{8}$$

$$\Rightarrow -\frac{h_\mathrm{B}}{Q_\mathrm{noVMI}}\left(Q_\mathrm{noVMI}-b\right)+\frac{\hat{\pi}b}{Q_\mathrm{noVMI}}+\frac{\pi D}{Q_\mathrm{noVMI}}=0 \tag{9}$$

$$\Rightarrow -h_\mathrm{B}+\frac{h_\mathrm{B}b}{Q_\mathrm{noVMI}}+\frac{\hat{\pi}b}{Q_\mathrm{noVMI}}+\frac{\pi D}{Q_\mathrm{noVMI}}=0 \tag{10}$$

$$\Rightarrow \frac{1}{Q_\mathrm{noVMI}}\left(h_\mathrm{B}b+\hat{\pi}b+\pi D\right)=h_\mathrm{B} \tag{11}$$

$$\Rightarrow Q_\mathrm{noVMI}=\frac{b(h_\mathrm{B}+\hat{\pi})}{h_\mathrm{B}}+\frac{\pi D}{h_\mathrm{B}} \tag{12}$$

$$\Rightarrow Q_\mathrm{noVMI}=\frac{\pi D}{h_\mathrm{B}}+b\left(1+\frac{\hat{\pi}}{h_\mathrm{B}}\right) \tag{13}$$

Inserting Eq. 13 into Eq. 7, we can write

$$\frac{1}{2}\left[\frac{\pi D}{h_\mathrm{B}}+b\left(1+\frac{\hat{\pi}}{h_\mathrm{B}}\right)\right]^2=\frac{1}{h_\mathrm{B}}\left(DA_\mathrm{B}+\pi Db+\frac{1}{2}\hat{\pi}b^2\right)+\frac{1}{2}b^2 \tag{14}$$

$$\Rightarrow \left(\hat{\pi}^2+\hat{\pi}h_\mathrm{B}\right)b^2+2\pi\hat{\pi}Db+(\pi D)^2-2DA_\mathrm{B}h_\mathrm{B}=0 \tag{15}$$

Since shortage is allowed and backlogged, i.e., $\hat{\pi}\neq 0$ and $\pi=0$, the optimal value of b can be obtained using Eq. 15 as b^*:

$$\left(\hat{\pi}^2+\hat{\pi}h_\mathrm{B}\right)b^2-2DA_\mathrm{B}h_\mathrm{B}=0 \tag{16}$$

$$\Rightarrow b^*=\sqrt{\frac{2DA_\mathrm{B}h_\mathrm{B}}{\hat{\pi}^2+\hat{\pi}h_\mathrm{B}}} \tag{17}$$

$$\Rightarrow b^*=\sqrt{\frac{2DA_\mathrm{B}h_\mathrm{B}}{\hat{\pi}(\hat{\pi}+h_\mathrm{B})}} \tag{18}$$

Inserting Eq. 18 into Eq. 13, we can obtain the optimal order quantity of the non-VMI supply chain as:

$$Q^*_\mathrm{noVMI}=\sqrt{\frac{2DA_\mathrm{B}h_\mathrm{B}}{\hat{\pi}(\hat{\pi}+h_\mathrm{B})}}\left(1+\frac{\hat{\pi}}{h_\mathrm{B}}\right) \tag{19}$$

$$=\sqrt{\frac{2DA_\mathrm{B}h_\mathrm{B}}{\hat{\pi}(\hat{\pi}+h_\mathrm{B})}\cdot\frac{(\hat{\pi}+h_\mathrm{B})^2}{h_\mathrm{B}^2}} \tag{20}$$

$$\Rightarrow Q^*_\mathrm{noVMI}=\sqrt{\left(\frac{2DA_\mathrm{B}}{h_\mathrm{B}}\right)\left(\frac{h_\mathrm{B}+\hat{\pi}}{\hat{\pi}}\right)} \tag{21}$$

Furthermore, inserting Eqs. 18 and 21 into Eq. 3, the total inventory cost of the supply chain before implementing VMI becomes:

$$\mathrm{TC}_\mathrm{noVMI}=\frac{1}{Q^*_\mathrm{noVMI}}\left[DA_\mathrm{B}+\frac{h_\mathrm{B}}{2}\left(Q^*_\mathrm{noVMI}-b^*\right)^2+\frac{\hat{\pi}b^{*2}}{2}+A_\mathrm{S}D\right] \tag{22}$$

$$=\frac{1}{\sqrt{\left(\frac{2DA_\mathrm{B}}{h_\mathrm{B}}\right)\left(\frac{h_\mathrm{B}+\hat{\pi}}{\hat{\pi}}\right)}}\left[\begin{array}{l}DA_\mathrm{B}+\frac{h_\mathrm{B}}{2}\left(\frac{2DA_\mathrm{B}}{h_\mathrm{B}}\left(1+\frac{h_\mathrm{B}}{\hat{\pi}+h_\mathrm{B}}\right)\right)\\ \quad+\frac{\hat{\pi}}{2}\left(\frac{2DA_\mathrm{B}h_\mathrm{B}}{\hat{\pi}(\hat{\pi}+h_\mathrm{B})}\right)+A_\mathrm{S}D\end{array}\right] \tag{23}$$

$$=\frac{1}{\sqrt{\left(\frac{2DA_\mathrm{B}}{h_\mathrm{B}}\right)\left(\frac{h_\mathrm{B}+\hat{\pi}}{\hat{\pi}}\right)}}\left[DA_\mathrm{B}+DA_\mathrm{B}\left(1+\frac{h_\mathrm{B}}{\hat{\pi}+h_\mathrm{B}}\right)+\frac{DA_\mathrm{B}h_\mathrm{B}}{\hat{\pi}+h_\mathrm{B}}+DA_\mathrm{S}\right] \tag{24}$$

$$=\frac{1}{\sqrt{\left(\frac{2DA_\mathrm{B}}{h_\mathrm{B}}\right)\left(\frac{h_\mathrm{B}+\hat{\pi}}{\hat{\pi}}\right)}}\left[DA_\mathrm{B}\left(1+1+\frac{h_\mathrm{B}}{\hat{\pi}+h_\mathrm{B}}+\frac{h_\mathrm{B}}{\hat{\pi}+h_\mathrm{B}}\right)+DA_\mathrm{S}\right] \tag{25}$$

$$\Rightarrow \mathrm{TC}_\mathrm{noVMI}=\frac{1}{\sqrt{\left(\frac{2DA_\mathrm{B}}{h_\mathrm{B}}\right)\left(\frac{h_\mathrm{B}+\hat{\pi}}{\hat{\pi}}\right)}}\left[2DA_\mathrm{B}\left(1+\frac{h_\mathrm{B}}{\hat{\pi}+h_\mathrm{B}}\right)+DA_\mathrm{S}\right] \tag{26}$$

4.4 The case of VMI (integrated) supply chain

After the implementation of VMI, the inventory costs of both the buyer and the supplier and, hence, the total inventory costs of the integrated supply chain are calculated as follows:

$$\mathrm{KB}_1 = 0 \tag{27}$$

$$\mathrm{KS}_1 = \frac{A_S D}{Q_{VMI}} + \frac{A_B D}{Q_{VMI}} + \frac{h_B}{2Q_{VMI}}(Q_{VMI} - b)^2 + \frac{\hat{\pi} b^2}{2Q_{VMI}} + \frac{\pi b D}{Q_{VMI}} \tag{28}$$

$$\mathrm{TC}_{VMI} = \mathrm{KB}_1 + \mathrm{KS}_1 = \frac{D}{Q_{VMI}}(A_B + A_S) + \frac{h_B}{2Q_{VMI}}(Q_{VMI} - b)^2 + \frac{\hat{\pi} b^2}{2Q_{VMI}} + \frac{\pi b D}{Q_{VMI}} \tag{29}$$

Once again, the total inventory cost of the integrated supply chain in Eq. 29 is a function of both Q_{VMI} and b for which the optimal values can be obtained by partial derivatives. Differentiating Eq. 29 with respect to Q_{VMI} and setting it zero results in:

$$\frac{\partial \mathrm{TC}_{VMI}}{\partial Q_{VMI}} = 0 \tag{30}$$

$$\Rightarrow -\frac{D}{Q_{VMI}^{*2}}(A_B + A_S) - \frac{h_B}{2Q_{VMI}^{*2}}(Q_{VMI} - b)^2 + \frac{h_B}{Q_{VMI}^{*}}\left(Q_{VMI}^{*} - b\right) - \frac{\hat{\pi} b^2}{2Q_{VMI}^{*2}} - \frac{\pi b D}{Q_{VMI}^{*2}} = 0 \tag{31}$$

$$\Rightarrow -\frac{1}{Q_{VMI}^{*2}}\left[D(A_B + A_S) + \frac{h_B}{2}\left(Q_{VMI}^{*} - b\right)^2 + \frac{\hat{\pi} b^2}{2} + \pi b D\right] + \frac{h_B}{Q_{VMI}^{*}}\left(Q_{VMI}^{*} - b\right) = 0 \tag{32}$$

$$\Rightarrow \left(-\frac{1}{Q_{VMI}^{*2}}\right)\frac{h_B}{2}\left(Q_{VMI}^{*} - b\right)^2 - \frac{1}{Q_{VMI}^{*2}}\left[D(A_B + A_S) + \frac{\hat{\pi} b^2}{2} + \pi b D\right] + \frac{h_B}{Q_{VMI}^{*}}\left(Q_{VMI}^{*} - b\right) = 0 \tag{33}$$

$$\Rightarrow \left(-\frac{1}{Q_{VMI}^{*2}}\right)\frac{h_B}{2}\left(Q_{VMI}^{*} - b\right)^2 + \frac{h_B}{Q_{VMI}^{*}}(Q_{VMI} - b) = \frac{1}{Q_{VMI}^{*2}}\left[D(A_B + A_S) + \frac{\hat{\pi} b^2}{2} + \pi b D\right] \tag{34}$$

$$\Rightarrow -\frac{1}{2}\left(Q_{VMI}^{*} - b\right)^2 + Q_{VMI}^{*}\left(Q_{VMI}^{*} - b\right) = \frac{1}{h_B}\left[D(A_B + A_S) + \frac{\hat{\pi} b^2}{2} + \pi b D\right] \tag{35}$$

$$\Rightarrow \left(Q_{VMI}^{*} - b\right)\left[-\frac{1}{2}\left(Q_{VMI}^{*} - b\right) + Q_{VMI}^{*}\right] = \frac{1}{h_B}\left[D(A_B + A_S) + \frac{\hat{\pi} b^2}{2} + \pi b D\right] \tag{36}$$

$$\Rightarrow \left(Q_{VMI}^{*} - b\right)\left[\frac{1}{2}\left(Q_{VMI}^{*} + b\right)\right] = \frac{1}{h_B}\left[D(A_B + A_S) + \frac{\hat{\pi} b^2}{2} + \pi b D\right] \tag{37}$$

$$\Rightarrow \frac{1}{2}\left(Q_{VMI}^{*2} - b^2\right) = \frac{1}{h_B}\left[D(A_B + A_S) + \frac{\hat{\pi} b^2}{2} + \pi b D\right] \tag{38}$$

$$\Rightarrow \frac{1}{2}Q_{VMI}^{*2} = \frac{1}{h_B}\left[D(A_B + A_S) + \frac{\hat{\pi} b^2}{2} + \pi b D\right] + \frac{1}{2}b^2 \tag{39}$$

Differentiating Eq. 29 with respect to b and setting it zero results in:

$$\frac{\partial \mathrm{TC}_{VMI}}{\partial b} = 0 \tag{40}$$

$$\Rightarrow -\frac{h_B}{Q_{VMI}}\left(Q_{VMI} - b^{*}\right) + \frac{1}{Q_{VMI}}\hat{\pi} b^{*} + \frac{\pi D}{Q_{VMI}} = 0 \tag{41}$$

$$\Rightarrow -h_B + \frac{h_B b^{*}}{Q_{VMI}} + \frac{1}{Q_{VMI}}\hat{\pi} b^{*} + \frac{\pi D}{Q_{VMI}} = 0 \tag{42}$$

$$\Rightarrow \frac{1}{Q_{VMI}}\left(h_B b^{*} + \hat{\pi} b^{*} + \pi D\right) = h_B \tag{43}$$

$$\Rightarrow Q_{VMI} = \frac{b^{*}(h_B + \hat{\pi})}{h_B} + \frac{\pi D}{h_B} \tag{44}$$

$$\Rightarrow Q_{VMI} = \frac{\pi D}{h_B} + b^{*}\left(1 + \frac{\hat{\pi}}{h_B}\right) \tag{45}$$

Inserting Eq. 45 into Eq. 39, we can write:

$$\frac{1}{2}\left[\frac{\pi D}{h_B}+b^*\left(1+\frac{\hat{\pi}}{h_B}\right)\right]^2=\frac{1}{h_B}\left[D(A_B+A_S)+\frac{\hat{\pi}b^2}{2}+\pi bD\right]+\frac{1}{2}b^{*2} \tag{46}$$

$$\Rightarrow\left(\hat{\pi}^2+\hat{\pi}h_B\right)b^{*2}+2\pi\hat{\pi}Db^*+(\pi D)^2-2D(A_B+A_S)h_B=0 \tag{47}$$

Considering $\hat{\pi}\neq 0$ and π=0 in Eq. 47 the optimal value of b^* is obtained as:

$$\left(\hat{\pi}^2+\hat{\pi}h_B\right)b^{*2}-2Dh_B(A_B+A_S)=0 \tag{48}$$

$$\Rightarrow b^*=\sqrt{\frac{2Dh_B(A_B+A_S)}{\hat{\pi}^2+\hat{\pi}h_B}} \tag{49}$$

$$\Rightarrow b^*=\sqrt{\frac{2Dh_B(A_B+A_S)}{\hat{\pi}(\hat{\pi}+h_B)}} \tag{50}$$

Therefore, by inserting Eq. 50 into Eq. 45 the optimal value of Q^*_{VMI} is obtained as:

$$Q^*_{VMI}=\sqrt{\frac{2Dh_B(A_B+A_S)}{\hat{\pi}(\hat{\pi}+h_B)}}\left(1+\frac{\hat{\pi}}{h_B}\right) \tag{51}$$

$$=\sqrt{\frac{2Dh_B(A_B+A_S)}{\hat{\pi}(\hat{\pi}+h_B)}\frac{(\hat{\pi}+h_B)^2}{h_B^2}} \tag{52}$$

$$\Rightarrow Q^*_{VMI}=\sqrt{\frac{2D(A_B+A_S)}{h_B}\left(\frac{\hat{\pi}+h_B}{\hat{\pi}}\right)} \tag{53}$$

Finally, by inserting Eqs. 50 and 53 into Eq. 29, the total inventory cost of the supply chain under VMI becomes:

$$TC_{VMI}=\frac{1}{Q^*_{VMI}}\left[D(A_B+A_S)+\frac{h_B}{2}\left(Q^*_{VMI}-b^*\right)^2+\frac{\hat{\pi}b^{*2}}{2}\right] \tag{54}$$

However, we need to obtain the term $\left(Q^*_{VMI}-b^*\right)$ as follows:

$$Q^*_{VMI}-b^*=\sqrt{\frac{2D(A_B+A_S)}{h_B}\left(\frac{\hat{\pi}+h_B}{\hat{\pi}}\right)}-\sqrt{\frac{2Dh_B(A_B+A_S)}{\hat{\pi}(\hat{\pi}+h_B)}} \tag{55}$$

$$=\sqrt{\frac{2D(A_B+A_S)}{h_B}\left(\frac{\hat{\pi}+h_B}{\hat{\pi}}\right)}-\sqrt{\frac{2Dh_B^2(A_B+A_S)}{\hat{\pi}(\hat{\pi}+h_B)h_B}} \tag{56}$$

$$=\sqrt{\frac{2D(A_B+A_S)}{h_B}}\left(\sqrt{\frac{\hat{\pi}+h_B}{\hat{\pi}}}-\sqrt{\frac{h_B^2}{\hat{\pi}(\hat{\pi}+h_B)}}\right) \tag{57}$$

$$=\sqrt{\frac{2D(A_B+A_S)}{h_B}\left(\frac{(\hat{\pi}+h_B)^2-h_B^2}{\hat{\pi}(\hat{\pi}+h_B)}\right)} \tag{58}$$

$$=\sqrt{\frac{2D(A_B+A_S)}{h_B}\left(\frac{\hat{\pi}^2+h_B^2+2\hat{\pi}h_B-h_B^2}{\hat{\pi}(\hat{\pi}+h_B)}\right)} \tag{59}$$

$$=\sqrt{\frac{2D(A_B+A_S)}{h_B}\left(\frac{\hat{\pi}(\hat{\pi}+2h_B)}{\hat{\pi}(\hat{\pi}+h_B)}\right)} \tag{60}$$

$$=\sqrt{\frac{2D(A_B+A_S)}{h_B}\left(\frac{\hat{\pi}+h_B}{\hat{\pi}+h_B}+\frac{h_B}{\hat{\pi}+h_B}\right)} \tag{61}$$

$$\Rightarrow Q^*_{VMI}-b^*=\sqrt{\frac{2D(A_B+A_S)}{h_B}\left(1+\frac{h_B}{\hat{\pi}+h_B}\right)} \tag{62}$$

Now, by inserting Eq. 62 into Eq. 54, we can write:

$$TC_{VMI}=\frac{D(A_B+A_S)+\frac{h_B}{2}\left(\sqrt{\frac{2D(A_B+A_S)}{h_B}\left(1+\frac{h_B}{\hat{\pi}+h_B}\right)}\right)^2+\frac{\hat{\pi}}{2}\left(\sqrt{\frac{2Dh_B(A_B+A_S)}{\hat{\pi}(\hat{\pi}+h_B)}}\right)^2}{\sqrt{\frac{2D(A_B+A_S)}{h_B}\left(\frac{\hat{\pi}+h_B}{\hat{\pi}}\right)}} \tag{63}$$

$$= \frac{D(A_{\mathrm{B}}+A_{\mathrm{S}})+D(A_{\mathrm{B}}+A_{\mathrm{S}})\left(1+\frac{h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)+D(A_{\mathrm{B}}+A_{\mathrm{S}})\left(\frac{h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)}{\sqrt{\frac{2D(A_{\mathrm{B}}+A_{\mathrm{S}})}{h_{\mathrm{B}}}\left(\frac{\hat{\pi}+h_{\mathrm{B}}}{\hat{\pi}}\right)}} \tag{64}$$

$$= \frac{D(A_{\mathrm{B}}+A_{\mathrm{S}})\left(1+1+\frac{h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}+\frac{h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)}{\sqrt{\frac{2D(A_{\mathrm{B}}+A_{\mathrm{S}})}{h_{\mathrm{B}}}\left(\frac{\hat{\pi}+h_{\mathrm{B}}}{\hat{\pi}}\right)}} \tag{65}$$

$$\Rightarrow \mathrm{TC}_{\mathrm{VMI}} = \frac{2D(A_{\mathrm{B}}+A_{\mathrm{S}})\left(1+\frac{h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)}{\sqrt{\frac{2D(A_{\mathrm{B}}+A_{\mathrm{S}})}{h_{\mathrm{B}}}\left(\frac{\hat{\pi}+h_{\mathrm{B}}}{\hat{\pi}}\right)}} \tag{66}$$

5 The analysis of inventory costs with and without VMI

In order to compare the total inventory costs of the supply chain in two cases of non-VMI and VMI implementation, let $\mathrm{TC}_{\mathrm{VMI}} \leq \mathrm{TC}_{\mathrm{noVMI}}$. Then, in what follows, a suitable requirement is derived on which this assumption holds.

$$\mathrm{TC}_{\mathrm{VMI}} \leq \mathrm{TC}_{\mathrm{noVMI}} \tag{67}$$

$$\Rightarrow \frac{\left[2D(A_{\mathrm{B}}+A_{\mathrm{S}})\left(1+\frac{h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)\right]}{\sqrt{\frac{2D(A_{\mathrm{B}}+A_{\mathrm{S}})}{h_{\mathrm{B}}}\left(\frac{\hat{\pi}+h_{\mathrm{B}}}{\hat{\pi}}\right)}} \leq \frac{\left[2DA_{\mathrm{B}}\left(1+\frac{h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)+DA_{\mathrm{S}}\right]}{\sqrt{\frac{2DA_{\mathrm{B}}}{h_{\mathrm{B}}}\left(\frac{h_{\mathrm{B}}+\hat{\pi}}{\hat{\pi}}\right)}} \tag{68}$$

$$\Rightarrow \frac{\left[2(A_{\mathrm{B}}+A_{\mathrm{S}})\left(1+\frac{h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)\right]}{\sqrt{\frac{2D(A_{\mathrm{B}}+A_{\mathrm{S}})}{h_{\mathrm{B}}}}} \leq \frac{\left[2A_{\mathrm{B}}\left(1+\frac{h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)+A_{\mathrm{S}}\right]}{\sqrt{\frac{2DA_{\mathrm{B}}}{h_{\mathrm{B}}}}} \tag{69}$$

$$\Rightarrow \frac{4(A_{\mathrm{B}}+A_{\mathrm{S}})^2\left(1+\frac{h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)^2}{\frac{2D(A_{\mathrm{B}}+A_{\mathrm{S}})}{h_{\mathrm{B}}}} \leq \frac{\left[4A_{\mathrm{B}}^2\left(1+\frac{h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)^2+A_{\mathrm{S}}^2+4A_{\mathrm{B}}A_{\mathrm{S}}\left(1+\frac{h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)\right]}{\frac{2DA_{\mathrm{B}}}{h_{\mathrm{B}}}} \tag{70}$$

$$\Rightarrow \frac{2h_{\mathrm{B}}(A_{\mathrm{B}}+A_{\mathrm{S}})\left(1+\frac{h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)^2}{D} \leq \frac{h_{\mathrm{B}}\left[4A_{\mathrm{B}}^2\left(1+\frac{h_{\mathrm{B}}^2}{(\hat{\pi}+h_{\mathrm{B}})^2}+\frac{2h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)+A_{\mathrm{S}}^2+4A_{\mathrm{B}}A_{\mathrm{S}}\left(1+\frac{h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)\right]}{2DA_{\mathrm{B}}} \tag{71}$$

$$\Rightarrow \frac{h_{\mathrm{B}}}{2DA_{\mathrm{B}}}\left[4A_{\mathrm{B}}^2+\frac{4A_{\mathrm{B}}^2h_{\mathrm{B}}^2}{(\hat{\pi}+h_{\mathrm{B}})^2}+\frac{8A_{\mathrm{B}}^2h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}+A_{\mathrm{S}}^2+4A_{\mathrm{B}}A_{\mathrm{S}}+\frac{4A_{\mathrm{B}}A_{\mathrm{S}}h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right] - \left[\frac{2h_{\mathrm{B}}(A_{\mathrm{B}}+A_{\mathrm{S}})}{D}\left(1+\frac{h_{\mathrm{B}}^2}{(\hat{\pi}+h_{\mathrm{B}})^2}+\frac{2h_{\mathrm{B}}}{\hat{\pi}+h_{\mathrm{B}}}\right)\right] \geq o \tag{72}$$

$$\Rightarrow \frac{4A_{\mathrm{B}}^2h_{\mathrm{B}}}{2DA_{\mathrm{B}}}+\frac{4A_{\mathrm{B}}^2h_{\mathrm{B}}^3}{2DA_{\mathrm{B}}(\hat{\pi}+h_{\mathrm{B}})^2}+\frac{8A_{\mathrm{B}}^2h_{\mathrm{B}}^2}{2DA_{\mathrm{B}}(\hat{\pi}+h_{\mathrm{B}})}+\frac{A_{\mathrm{S}}^2h_{\mathrm{B}}}{2DA_{\mathrm{B}}}+\frac{4A_{\mathrm{B}}A_{\mathrm{S}}h_{\mathrm{B}}}{2DA_{\mathrm{B}}} +\frac{4A_{\mathrm{B}}A_{\mathrm{S}}h_{\mathrm{B}}^2}{2DA_{\mathrm{B}}(\hat{\pi}+h_{\mathrm{B}})}-\frac{2h_{\mathrm{B}}(A_{\mathrm{B}}+A_{\mathrm{S}})}{D}-\frac{2h_{\mathrm{B}}^3(A_{\mathrm{B}}+A_{\mathrm{S}})}{D(\hat{\pi}+h_{\mathrm{B}})^2} -\frac{4h_{\mathrm{B}}^2(A_{\mathrm{B}}+A_{\mathrm{S}})}{D(\hat{\pi}+h_{\mathrm{B}})} \geq o \tag{73}$$

$$\Rightarrow \frac{2A_{\mathrm{B}}h_{\mathrm{B}}}{D}+\frac{2A_{\mathrm{B}}h_{\mathrm{B}}^3}{D(\hat{\pi}+h_{\mathrm{B}})^2}+\frac{4A_{\mathrm{B}}h_{\mathrm{B}}^2}{D(\hat{\pi}+h_{\mathrm{B}})}+\frac{A_{\mathrm{S}}^2h_{\mathrm{B}}}{2DA_{\mathrm{B}}}+\frac{2A_{\mathrm{S}}h_{\mathrm{B}}}{D}+\frac{2A_{\mathrm{S}}h_{\mathrm{B}}^2}{D(\hat{\pi}+h_{\mathrm{B}})} -\frac{2h_{\mathrm{B}}(A_{\mathrm{B}}+A_{\mathrm{S}})}{D}-\frac{2h_{\mathrm{B}}^3(A_{\mathrm{B}}+A_{\mathrm{S}})}{D(\hat{\pi}+h_{\mathrm{B}})^2}-\frac{4h_{\mathrm{B}}^2(A_{\mathrm{B}}+A_{\mathrm{S}})}{D(\hat{\pi}+h_{\mathrm{B}})} \geq o \tag{74}$$

$$\Rightarrow \left[\frac{2A_{\mathrm{B}}h_{\mathrm{B}}^3-2h_{\mathrm{B}}^3(A_{\mathrm{B}}+A_{\mathrm{S}})}{D(\hat{\pi}+h_{\mathrm{B}})^2}\right]+\left[\frac{4A_{\mathrm{B}}h_{\mathrm{B}}^2+2A_{\mathrm{S}}h_{\mathrm{B}}^2-4h_{\mathrm{B}}^2A_{\mathrm{B}}-4h_{\mathrm{B}}^2A_{\mathrm{S}}}{D(\hat{\pi}+h_{\mathrm{B}})}\right] +\left[\frac{2A_{\mathrm{B}}h_{\mathrm{B}}+2A_{\mathrm{S}}h_{\mathrm{B}}-2h_{\mathrm{B}}A_{\mathrm{B}}-2h_{\mathrm{B}}A_{\mathrm{S}}}{D}\right]+\frac{A_{\mathrm{S}}^2h_{\mathrm{B}}}{2DA_{\mathrm{B}}} \geq o \tag{75}$$

$$\Rightarrow \frac{-2A_{\mathrm{S}}h_{\mathrm{B}}^3}{D(\hat{\pi}+h_{\mathrm{B}})^2}-\frac{2A_{\mathrm{S}}h_{\mathrm{B}}^2}{D(\hat{\pi}+h_{\mathrm{B}})}+\frac{A_{\mathrm{S}}^2h_{\mathrm{B}}}{2DA_{\mathrm{B}}} \geq o \tag{76}$$

$$\Rightarrow \frac{A_{\mathrm{S}}h_{\mathrm{B}}}{D}\left[\frac{-2h_{\mathrm{B}}^2}{(\hat{\pi}+h_{\mathrm{B}})^2}-\frac{2h_{\mathrm{B}}}{(\hat{\pi}+h_{\mathrm{B}})}+\frac{A_{\mathrm{S}}}{2A_{\mathrm{B}}}\right] \geq o \tag{77}$$

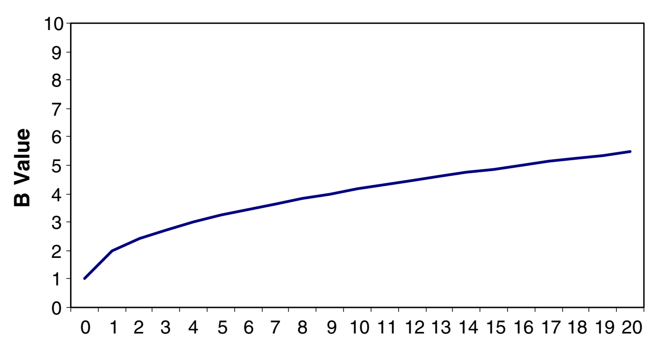


Fig. 4 The VMI to non-VMI ratio of the supplier's order quantity vs d

Table 1 The initial data of the numerical examples

The parameters	Numerical examples		
	Example 1	Example 2	Example 3
A_B	21	30	45
A_S	75	40	150
h_B	90	90	90
D	8,000	8,000	8,000
$\hat{\pi}$	80	80	80

$$\Rightarrow \frac{A_S h_B}{D}\left[\frac{-4A_B h_B^2 - 4A_B h_B(\hat{\pi}+h_B) + A_S(\hat{\pi}+h_B)^2}{2A_B(\hat{\pi}+h_B)^2}\right] \geq o \tag{78}$$

$$\Rightarrow \frac{A_S h_B}{2A_B D(\hat{\pi}+h_B)^2}\left[-4A_B h_B^2 - 4A_B h_B\hat{\pi} - 4A_B h_B^2 + A_S(\hat{\pi}+h_B)^2\right] \geq o \tag{79}$$

$$\Rightarrow \frac{A_S h_B}{2A_B D(\hat{\pi}+h_B)^2}\left[-8A_B h_B^2 - 4A_B h_B\hat{\pi} + A_S(\hat{\pi}+h_B)^2\right] \geq o \tag{80}$$

The term $\frac{A_S h_B}{2A_B D\left(\hat{\pi}+h_B\right)^2}$ is always positive, therefore:

$$\left[-8A_B h_B^2 - 4A_B h_B\hat{\pi} + A_S(\hat{\pi}+h_B)^2\right] \geq o \tag{81}$$

$$\Rightarrow \left[-A_B h_B(8h_B + 4\hat{\pi}) + A_S(\hat{\pi}+h_B)^2\right] \geq o \tag{82}$$

$$\Rightarrow A_S(\hat{\pi}+h_B)^2 \geq A_B h_B(8h_B + 4\hat{\pi}) \tag{83}$$

$$\Rightarrow A_S(\hat{\pi}+h_B)^2 \geq 4A_B h_B(2h_B + \hat{\pi}) \tag{84}$$

$$\Rightarrow A_S \geq A_B\left[\frac{4h_B(2h_B+\hat{\pi})}{(\hat{\pi}+h_B)^2}\right] \tag{85}$$

Table 2 The results obtained for example 1

	Non-VMI supply chain	VMI-supply chain	α	Does Eq. 85 hold?	The selected chain
b^*	1.47	100.82	68.01	Yes	VMI
Q^*	89.07	190.44			
Total cost	12,505.80	12,335.64			

Table 3 The results obtained for example 2

	Non-VMI supply chain	VMI-supply chain	α	Does Eq. 85 hold?	The selected chain
b^*	1.50	100.82	97.16	No	Non-VMI
Q^*	106.46	190.44			
Total cost	9,901.71	10,533.56			

Hence, the following conclusion can be made in a single-buyer–single-supplier supply chain system where the EOQ model is used and shortages are completely backlogged:

- The VMI integrated supply system is not always better than the traditional one.
- The VMI implementation has the ability to reduce the total costs of the supply chain only when Eq. 85 holds.

5.1 The VMI-to-non-VMI ratio of supplier's order quantity

Let β be the VMI-to-non-VMI ratio of the supplier's order quantity, i.e.,

$$\beta = \frac{Q_{VMI}}{Q_{noVMI}} \tag{86}$$

Then,

$$\beta = \frac{\sqrt{\frac{2D(A_B+A_S)}{h_B}\left(\frac{\hat{\pi}+h_B}{\hat{\pi}}\right)}}{\sqrt{\frac{2DA_B}{h_B}\left(\frac{\hat{\pi}+h_B}{\hat{\pi}}\right)}} = \frac{\sqrt{A_B+A_S}}{\sqrt{A_B}} = 1 + \sqrt{\frac{A_S}{A_B}} = 1 + \sqrt{d}, \tag{87}$$

where $d = \frac{A_S}{A_B}$. Since the quantity of β monotonically increases with respect to d, the following conclusions can then be made:

(a) The ratio of the VMI order quantity to non-VMI order quantity (β) monotonically increases with respect to $d = \frac{A_S}{A_B}$. Figure 4 depicts this conclusion.
(b) The larger the ratio of the supplier to the retailer order cost, the larger the ratio of VMI to non-VMI order quantity is.

Table 4 The results obtained for example 3

	Non-VMI supply chain	VMI-supply chain	α	Does Eq. 85 hold?	The selected chain
b^*	1.53	143.69	145.74	Yes	VMI
Q^*	130.38	271.42			
Total cost	17,649.22	17,581.00			

(c) Since small values of d cause the retailer' order cost to be larger than the supplier's order cost and since $\beta = 1 + \sqrt{d}$, the implementation of VMI reduces order quantities by a greater amount when the order cost ratio is small. This is a favor for the supplier because its order cost decreases.

6 Numerical examples

In this section, three numerical examples are given to justify and demonstrate the application of the proposed method. In these examples, let $\alpha = A_B \left[\frac{4h_B(2h_B + \hat{\pi})}{(\hat{\pi} + h_B)^2} \right]$. Hence, based on Eq. 85, if the inequality $A_S \geq \alpha$ holds, the total costs of the VMI system will be smaller than the one of the non-VMI supply chain.

The initial data of the numerical examples are given in Table 1, in which the examples share equal demand, equal carrying cost, and equal shortage cost. However, different buyer and supplier's ordering costs are used. The results of applying the proposed method on the data of the numerical examples are given in Tables 2, 3, and 4.

The results of Tables 2, 3, and 4 show that employing Eq. 85 is justified and the supply chain with less total cost is selected as the better one in all of the three examples.

7 Conclusions and future research

This paper contributes to the literature an analytical model that helps to provide a better understanding of how important supply chain parameters affect the inventory cost savings to be realized from VMI implementation. This model was considered for a two-level supply chain, which consisted of a single supplier and a single buyer by which the inventory management practices before and after the implementation of VMI were examined. Results from the model showed that VMI implementation in EOQ model, when shortage is backlogged, has the ability to reduce total costs of supply chain only in a special case (when Eq. 85 for A_S and A_B holds). In other words, we showed that cost savings earned by implementing collaborative initiatives such as VMI that allows information sharing and integration among firms in the supply chain cannot be reached without limitation.

Since the proposed model of this research is representative of a certain type supply chain, the results are only true when the assumptions are held. In particular, the demand was assumed to be known with certainty, the lead time was assumed to be zero, and only one item was assumed to be in the supply chain. The study of changing the above assumptions is a possible topic for further research.

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